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Non-equilibrium critical relaxation with reversible mode coupling

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Abstract. The non-equilibrium critical relaxation of systems in which the order parameter couples reversibly to conserved densities is investigated. The initial state which may be macroscopically prepared by a quench from a temperature $T \gg T_c$ to the critical temperature T_c is characterized by short-range correlations. In the case where the order parameter itself is conserved the relaxation shows universal scaling. If, on the other hand, the order parameter is not conserved the scaling is governed by a new exponent which depends on the width of the initial distribution of the conserved fields. However, the response function of the conserved fields still shows universal scaling.

1. Introduction

The relaxation of a thermodynamic system after a quench from an initial temperature $T_0 \gg T_c$ to the critical temperature T_c is governed by the growth of correlations. Very recently we have studied the non-equilibrium critical relaxation of systems in which the order parameter couples to a conserved density [1]. Our study was based on model C (defined by Halperin *et al* [2]) which serves as a suitable description of the critical dynamics provided that no reversible mode coupling is present.

For model A and model C the correlation function $C(\boldsymbol{r}; t, t')$ and the linear response function $\chi(\boldsymbol{r}; t, t')$ display universal scaling behaviour [1, 3, 4]

$$C(\boldsymbol{r}; t, t') = r^{-(d-2+\eta)} \left(\frac{t}{t'}\right)^{\theta-1} f_C\left(\frac{r}{\xi}, \frac{t}{\xi^z}, \frac{t'}{t}\right) \tag{1}$$

$$\chi(\boldsymbol{r}; t, t') = r^{-(d-2+\eta+z)} \left(\frac{t}{t'}\right)^{\theta} f_\chi\left(\frac{r}{\xi}, \frac{t}{\xi^z}, \frac{t'}{t}\right) \tag{2}$$

(ξ denotes the equilibrium correlation length) where the scaling functions f_C and f_χ remain finite for $t' \to 0$, i.e. for short times t' after the quench the scaling behaviour is governed by the exponent θ. If the initial value $M(0) = M_0$ of the order parameter is non-zero it decays at T_c according to the scaling law

$$M(t) = M_0 t^{\theta'} f_M(t^{\theta'+\beta/(\nu z)} M_0) \tag{3}$$

with $\theta' = \theta + (2 - z - \eta)/z$ and

$$f_M(x) \sim \begin{cases} 1 & \text{for } x \to \infty \\ 1/x & \text{for } x \to 0\,. \end{cases} \tag{4}$$

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For model A and model C the exponent θ has been calculated to second order in $\epsilon = 4 - d$ (where d is the spatial dimension of the system).

In this paper we consider the effect of reversible mode coupling on the relaxation. Our results apply for example to planar (anti-)ferromagnets (model E [5]), isotropic antiferromagnets (model G), and the more general O(n)-symmetric model of Sasvári *et al* [6]. We show that the relaxation of these systems is subject to a strong influence of the initial fluctuations of the conserved fields coupling to the order parameter. This influence becomes apparent in a non-universal exponent θ which depends on the width of the initial distribution of the conserved densities. In section 4 we use dynamic field theory [7, 8, 9] to calculate θ to first order in $\epsilon = 4 - d$.

As an example of a system with conserved order parameter and reversible mode coupling we study in section 5 the isotropic Heisenberg ferromagnet (model J). In this case the initial slip exponent proves to be universal, but it is not new since it can be expressed in terms of η and z.

2. The model

Physical systems with continuous symmetries satisfy conservation laws which result from Noether's theorem. One of us [9] has introduced a model for the critical dynamics of a non-conserved n-component order parameter s which transforms according to an irreducible representation of a p-parametric internal symmetry group. The p infinitesimal generators m_i are conserved densities and thus belong to the set of slow variables which govern the infrared limit of correlation and response functions. The coupling of the conserved fields to the order parameter can be described by reversible contributions to the Langevin equations [9]

$$\partial_t s_\alpha = -\lambda \frac{\delta \mathcal{H}}{\delta s_\alpha} - M_{\alpha j}(s) \frac{\delta \mathcal{H}}{\delta m_j} + \zeta_\alpha \tag{5}$$

$$\partial_t m_j = \lambda \rho \Delta \frac{\delta \mathcal{H}}{\delta m_j} - M_{j\alpha}(s) \frac{\delta \mathcal{H}}{\delta s_\alpha} + \eta_j \tag{6}$$

where ζ and η are Gaussian random forces with zero mean and the correlations

$$\langle \zeta_\alpha(\boldsymbol{r}, t) \zeta_\beta(\boldsymbol{r}', t') \rangle = 2\lambda \delta_{\alpha\beta} \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(t - t') \tag{7}$$

$$\langle \eta_j(\boldsymbol{r}, t) \eta_k(\boldsymbol{r}', t') \rangle = -2\lambda\rho \delta_{jk} \Delta \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(t - t') \,. \tag{8}$$

The Hamiltonian reads

$$\mathcal{H}[s, m] = \int \mathrm{d}^d r \left[\frac{\tau}{2} s^2 + \frac{1}{2} (\nabla s)^2 + \frac{1}{4!} (g s^4) + \frac{1}{2} m^2 \right] \tag{9}$$

where (gs^4) is a sum of forth order invariants of the order parameter representation. Hereafter we set $(gs^4) = g(s^2)^2$, i.e. $\mathcal{H}$ is invariant with respect to O(n).

The relevant contributions to the antisymmetric mode coupling matrix $M(s)$ are

$$M_{j\alpha}(s) = -M_{\alpha j}(s) = \lambda f I_{j,\alpha\beta} s_\beta \,. \tag{10}$$

For simplicity we assume that the coefficients $I_{j,\alpha\beta}$ are uniquely determined by the symmetry of the system† and the conservation of m. In this case they can be derived from Poisson bracket relations (see, e.g., [10]).

† The $I_{j,\alpha\beta}$ are Clebsch–Gordan coefficients which reduce the direct product representation of the order parameter representation antisymmetrically to the adjoint representation.

An equivalent formulation for the dynamics is given by the stochastic functional [9]

$$\mathcal{J}[\tilde{s}, s; \tilde{m}, m] = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d r \, \{\tilde{s}[\partial_t s + \lambda(\tau - \Delta)s + (\lambda/3!)(gs^3)] - \lambda\tilde{s}^2$$
$$+ \tilde{m}(\partial_t m - \lambda\rho\Delta m) - \lambda\rho(\nabla\tilde{m})^2 - \lambda f[\tilde{s}(mI)s - (\nabla s)(\nabla\tilde{m}I)s]\} . \qquad (11)$$

where $\tilde{s}$ and $\tilde{m}$ are Martin–Siggia–Rose response fields [11].

The only second order invariant of an irreducible orthogonal representation is $s^2 = s_\alpha s_\alpha$. By an appropriate normalization of the $I_{i,\alpha\beta}$ we therefore achieve

$$I_{i,\alpha\gamma} I_{i,\beta\gamma} = p\delta_{\alpha\beta} \qquad I_{i,\alpha\beta} I_{j,\alpha\beta} = n\delta_{ij} . \qquad (12)$$

The model defined above includes the O(n)-symmetric model of Sasvári *et al* [6]. The special case $n = p = 3$ corresponds to the isotropic antiferromagnet (model G), and $n = 2$, $p = 1$ gives the planar (anti-) ferromagnet (model E).

The Langevin equations (5), (6) and the dynamic functional (11) describe the equilibrium critical dynamics of the model. The non-equilibrium initial state with short-range correlations corresponds to the distribution [1]

$$\mathcal{P}[s_0, m_0] \propto \exp(-\mathcal{H}^{(i)}[s_0, m_0]) \qquad (13)$$

where $s_0(\boldsymbol{r}) = s(\boldsymbol{r}, 0)$, $m_0(\boldsymbol{r}) = m(\boldsymbol{r}, 0)$, and

$$\mathcal{H}^{(i)}[s_0, m_0] = \int \mathrm{d}^d r \left[\frac{\tau_0}{2} s_0^2 + \frac{1}{2c_0} m_0^2\right] . \qquad (14)$$

Since τ_0^{-1} is an irrelevant variable [3] we may set $\tau_0 = \infty$ and use the sharp initial condition $s_0(\boldsymbol{r}) = 0$ for the order parameter field.

In order to obtain non-equilibrium correlation and response functions we have to average products of fields with respect to both thermal noise *and* initial conditions, i.e. integrals of the form

$$\langle s(\boldsymbol{r}, t) \cdots \rangle = \int \mathcal{D}[i\tilde{s}, s; i\tilde{m}, m] \, s(\boldsymbol{r}, t) \cdots \exp(-\mathcal{J}[\tilde{s}, s; \tilde{m}, m] - \mathcal{H}^{(i)}[s_0, m_0])$$

have to be calculated. The linear response to an external field h conjugate to the order parameter is given by

$$\chi_s(\boldsymbol{r}; t, t') = \frac{\delta\langle s(\boldsymbol{r}, t)\rangle}{\delta h(\boldsymbol{0}, t')} = \langle s(\boldsymbol{r}, t)\phi^{(s)}(\boldsymbol{0}, t')\rangle \qquad (15)$$

with the composite response field

$$\phi_\alpha^{(s)} = \lambda(\tilde{s}_\alpha + f\tilde{m}_j I_{j,\alpha\beta} s_\beta) . \qquad (16)$$

In section 4 we will also consider the response function for the conserved densities

$$\chi_m(\boldsymbol{r}; t, t') = \frac{\delta\langle m(\boldsymbol{r}, t)\rangle}{\delta h_m(\boldsymbol{0}, t')} = \langle m(\boldsymbol{r}, t)\phi^{(m)}(\boldsymbol{0}, t')\rangle \qquad (17)$$

where h_m is an external field conjugate to m and

$$\phi_j^{(m)} = -\lambda(\rho\Delta\tilde{m}_j + f I_{j,\alpha\beta}\tilde{s}_\alpha s_\beta) . \qquad (18)$$

3. Renormalization

We proceed along the lines taken in the analysis of the relaxation behaviour of model C. Methods of renormalized field theory will be used to obtain the scaling behaviour of non-equilibrium response and correlation functions.

The propagator and the correlator of the 'free' Gaussian model defined by $g = f = 0$ are given by

$$G_q(t,t') = \exp(-\lambda(\tau + q^2)(t - t')) \qquad \text{for } t > t' \tag{19}$$

$$C_q(t,t') = C_q^{(eq)}(t - t') + C_q^{(i)}(t,t') \tag{20}$$

with the equilibrium correlator

$$C_q^{(eq)}(t - t') = \frac{1}{\tau + q^2} \exp(-\lambda(\tau + q^2)|t - t'|) \tag{21}$$

and a non-equilibrium ('initial') part

$$C_q^{(i)}(t,t') = \left(\tau_0^{-1} - \frac{1}{\tau + q^2}\right) \exp(-\lambda(\tau + q^2)(t + t'))\,. \tag{22}$$

Hereafter we set $\tau_0 = \infty$, i.e. we impose Dirichlet initial conditions for the order parameter field s. The corresponding functions for the conserved fields are

$$G_{m,q}(t - t') = \exp(-\lambda\rho q^2(t - t')) \qquad \text{for } t > t' \tag{23}$$

$$C_{m,q}(t,t') = C_{m,q}^{(eq)}(t - t') + C_{m,q}^{(i)}(t,t') \tag{24}$$

where

$$C_{m,q}^{(eq)}(t - t') = \exp(-\lambda\rho q^2|t - t'|) \tag{25}$$

$$C_{m,q}^{(i)}(t,t') = (c_0 - 1)\exp(-\lambda\rho q^2(t + t'))\,. \tag{26}$$

A perturbative calculation of Green functions (treating the coupling coefficients f and g as perturbations) leads to integrals which are ultraviolet-divergent at the upper critical dimension $d_c = 4$. We render these integrals finite by analytic continuation in d (dimensional regularization) and absorb the remaining poles at $\epsilon = 0$ into reparametrizations of coupling coefficients and fields, i.e. we apply the minimal renormalization prescription.

The renormalizations necessary to render the equilibrium theory finite are [9]

$$\begin{aligned}
&s \to \mathring{s} = Z_s^{1/2} s && \tilde{s} \to \mathring{\tilde{s}} = Z_{\tilde{s}}^{1/2} \tilde{s} \\
&m \to \mathring{m} = Z_m^{1/2} m && \tilde{m} \to \mathring{\tilde{m}} = Z_m^{-1/2} \tilde{m} \\
&\tau \to \mathring{\tau} = Z_s^{-1} Z_\tau \tau && \\
&g \to \mathring{g} = Z_s^{-2} Z_u g && f \to \mathring{f} = (Z_{\tilde{s}}/Z_s)^{1/2} Z_v f \\
&\lambda \to \mathring{\lambda} = (Z_s/Z_{\tilde{s}})^{1/2} Z_\lambda \lambda && \rho \to \mathring{\rho} = (Z_{\tilde{s}}/Z_s)^{1/2} Z_\rho Z_m \rho \\
&G_\epsilon g = u\mu^\epsilon && G_\epsilon f^2 = v^2 \mu^\epsilon\,.
\end{aligned}$$

The geometrical factor $G_\epsilon = \Gamma(1 + \epsilon/2)/(4\pi)^{d/2}$ has been introduced for convenience, and μ is an external momentum scale. The renormalization factors Z_s and Z_u are the same as in model A. As a result of the purely reversible coupling of the conserved density to the order parameter the identity $Z_m = 1$ holds in every order of the perturbation theory.

Furthermore, a dissipation fluctuation theorem [9] yields $Z_\lambda Z_v = 1$. The remaining Z factors are given by

$$Z_{\tilde{s}} = 1 + \mathrm{O}(\text{two-loop}) \tag{27}$$

$$Z_\lambda = 1 - \frac{2pv^2}{(1+\rho)\epsilon} + \mathrm{O}(\text{two-loop}) \tag{28}$$

$$Z_\lambda Z_\rho = 1 - \frac{nv^2}{2\rho\epsilon} + \mathrm{O}(\text{two-loop})\,. \tag{29}$$

The non-equilibrium part of the correlator (20) generates additional divergences which have to be subtracted by a further renormalization of the initial field $\tilde{s}_0(\boldsymbol{r}) = \tilde{s}(\boldsymbol{r}, 0)$

$$\tilde{s}_0 \to \mathring{\tilde{s}}_0 = (Z_0 Z_{\tilde{s}})^{1/2} \tilde{s}_0\,. \tag{30}$$

To obtain the new Z factor Z_0 we split the Green function $\mathring{G}^1_{0,1}(t) = \langle \mathring{s}(t) \mathring{\tilde{s}}_0 \rangle$ into two parts

$$\mathring{G}^1_{0,1}(\boldsymbol{q}, t) = \int_0^t \mathrm{d}t'\, \mathring{G}^{(eq)}_{1,1}(\boldsymbol{q}; t - t') \mathring{\Gamma}_{1,0}(\boldsymbol{q}, t')_{[\mathring{\tilde{s}}_0]} \tag{31}$$

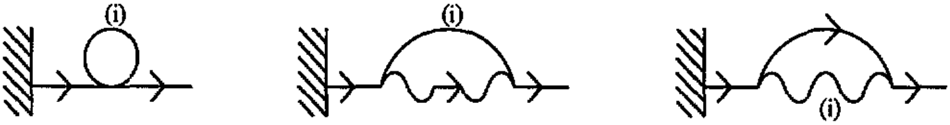

Figure 1. Contributions to $\Gamma_{1,0}(\boldsymbol{q}, t)_{[\tilde{s}_0]}$ at one loop order. The wavy lines are correlators and propagators of the conserved fields. Initial parts of correlators are indicated by the letter '(i)'. The hatched area represents the 'time surface' $t = 0$.

where $\mathring{G}^{(eq)}_{1,1} = \langle \mathring{s}(t) \mathring{\tilde{s}}(t') \rangle$ is a one-particle-reducible equilibrium Green function, and $\mathring{\Gamma}_{1,0}(\boldsymbol{q}, t)_{[\mathring{\tilde{s}}_0]}$ contains additional contributions to $\mathring{G}^1_{0,1}(\boldsymbol{q}, t)$ which result from the broken translational invariance with respect to time [1]. The Feynman graphs contributing to $\mathring{\Gamma}_{1,0}(\boldsymbol{q}, t)_{[\mathring{\tilde{s}}_0]}$ at one loop order are shown in figure 1.

We find (for $\tau = 0$) that

$$\begin{aligned}
\mathring{\Gamma}_{1,0}(\boldsymbol{0}, t)_{[\mathring{\tilde{s}}_0]} = \delta(t) &+ \frac{n+2}{6} \frac{\lambda G_\epsilon g}{1 - \epsilon/2} \frac{(2\lambda t)^{-1+\epsilon/2}}{\Gamma(1+\epsilon/2)} \\
&- \frac{p\lambda G_\epsilon f^2}{1-\epsilon/2} \frac{1}{\rho - 1} [2^{-1+\epsilon/2} - (1+\rho)^{-1+\epsilon/2}] \frac{(\lambda t)^{-1+\epsilon/2}}{\Gamma(1+\epsilon/2)} \\
&- (c_0 - 1) \frac{p\lambda G_\epsilon f^2}{1-\epsilon/2} \frac{1}{\rho - 1} [(1+\rho)^{-1+\epsilon/2} - (2\rho)^{-1+\epsilon/2}] \frac{(\lambda t)^{-1+\epsilon/2}}{\Gamma(1+\epsilon/2)} \\
&+ \mathrm{O}(\text{two-loop})\,.
\end{aligned} \tag{32}$$

To obtain a well defined renormalized Green function $G^1_{0,1} = (Z_0 Z_{\tilde{s}} Z_s)^{-1/2} \mathring{G}^1_{0,1}$ we have to render the Laplace transform

$$\int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}\omega t} \Gamma_{1,0}(\boldsymbol{0}, t)_{[\tilde{s}_0]} = Z_0^{-1/2} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}\omega t} \mathring{\Gamma}_{1,0}(\boldsymbol{0}, t)_{[\mathring{\tilde{s}}_0]} \tag{33}$$

finite for $\epsilon \to 0$. This yields

$$Z_0 = 1 + \frac{n+2}{3\epsilon}u - \frac{2pv^2}{\epsilon(1+\rho)} - (c_0 - 1)\frac{2pv^2}{\epsilon\rho(1+\rho)} + \mathrm{O(two\text{-}loop)}\,. \qquad (34)$$

The parameter c_0 requires no renormalization since

$$\int \mathrm{d}^d r\ \langle \mathring{m}(\boldsymbol{r}, t)\mathring{m}(\boldsymbol{0}, t')\rangle = c_0 \qquad \text{for } t, t' \geqslant 0 \qquad (35)$$

and $\mathring{m} = m$. Equation (35) follows from the conservation of m [1].

4. Scaling

In this section we study the scaling behaviour of Green functions

$$G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M} = \langle [\tilde{s}_0]^{\tilde{L}} [\tilde{s}]^{\tilde{N}} [s]^N [\tilde{m}]^{\tilde{M}} [m]^M \rangle \qquad (36)$$

with $\tilde{L}$ insertions of the field $\tilde{s}_0$. Renormalized Green functions are related to the coresponding bare functions by

$$G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M} \to \mathring{G}^{\tilde{L}}_{\tilde{N},N;\tilde{M},M} = (Z_0 Z_{\tilde{s}})^{\tilde{L}/2} Z_{\tilde{s}}^{\tilde{N}/2} Z_s^{N/2} G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M}\,. \qquad (37)$$

We exploit the μ-independence of the bare Green functions to derive the renormalization group equation (RGE)

$$\left[\mu\partial_\mu + \zeta\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta_u\partial_u + \beta_v\partial_v + \beta_\rho\partial_\rho + \frac{\tilde{L}}{2}(\gamma_0 + \gamma_{\tilde{s}}) + \frac{\tilde{N}}{2}\gamma_{\tilde{s}} + \frac{N}{2}\gamma_s\right] G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M} = 0 \qquad (38)$$

for the renormalized functions. Here $\beta_w = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\big|_0 w$ (for $w = u, v, \rho$) and $\gamma_\alpha = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\big|_0 \ln Z_\alpha$ (for $\alpha = s, \tilde{s}, \lambda, \rho, o$) are Wilson functions, where $\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\big|_0$ means a derivative at fixed bare parameters. The new Wilson function connected with the non-equilibrium initial conditions follows from (34)

$$\gamma_0 = -\frac{n+2}{3}u + \frac{2pv^2}{1+\rho}\left(1 + \frac{c_0 - 1}{\rho}\right) + \mathrm{O(two\text{-}loop)}\,. \qquad (39)$$

In a region $n < 4p + \mathrm{O}(\epsilon)$ in the (d, n, p)-space the dynamic scaling fixed point

$$v_*^2 = \epsilon/(4p - n) + \mathrm{O}(\epsilon^2) \qquad \rho_* = n/(4p - n) + \mathrm{O}(\epsilon) \qquad (40)$$

is stable [9]. Directly at this fixed point the Green functions display the scaling behaviour

$$G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M}(\{\boldsymbol{r}, t\}; \tau; \lambda, \mu) = l^{(d-2+\eta)N/2 + (d+2+\tilde{\eta})(\tilde{N}+\tilde{L})/2 + \eta_0\tilde{L}/2 + d(M+\tilde{M})/2} \times G^{\tilde{L}}_{\tilde{N},N;\tilde{M},M}(\{l\boldsymbol{r}, l^z t\}; l^{-1/\nu}\tau; \lambda, \mu)\,. \qquad (41)$$

with $z = d/2$ and

$$\eta_0 = \gamma_0^* = -\frac{n+2}{n+8}\epsilon + \left[1 + \left(\frac{4p}{n} - 1\right)(c_0 - 1)\right]\frac{\epsilon}{2} + \mathrm{O}(\epsilon^2)\,. \qquad (42)$$

Due to the marginality of the parameter c_0 the exponent η_0 is not universal but depends on the width of the initial distribution of the conserved densities.

The short-time scaling behaviour of correlation and response functions given in the introduction can be derived from the short time expansion [3]

$$\begin{aligned} s(t) &= \sigma(t)\dot{s}_0 + \cdots \\ \phi^{(s)}(t) &= \tilde{\sigma}(t)\phi_0^{(s)} + \cdots \end{aligned} \qquad \text{for } t \to 0 \tag{43}$$

which is valid after insertion into averages. An insertion of the time derivative $\dot{s}_0 = \partial_t s|_{t=0} = 2\lambda\tilde{s}_0$ (for $\tau_0 = \infty$) requires the renormalization

$$\dot{s}_0 \to \mathring{\dot{s}}_0 = (Z_0 Z_s)^{1/2} Z_\lambda \dot{s}_0 \,. \tag{44}$$

The renormalization group equation

$$[\mu\partial_\mu + \zeta\lambda\partial_\lambda + \beta_u\partial_u + \beta_v\partial_v + \beta_\rho\partial_\rho - \gamma_\lambda - \tfrac{1}{2}\gamma_0]\sigma(t; u, v, \rho; \lambda, \mu) = 0 \tag{45}$$

for the coefficient $\sigma(t)$ follows from the different renormalizations of $s(t)$ and $\dot{s}_0$. Equation (45) together with dimensional analysis gives at the fixed point (40) the power law

$$\sigma(t) \sim t(\mu^2\lambda t)^{(x_\lambda+\eta_0/2)/z} \tag{46}$$

with

$$x_\lambda = \gamma_\lambda^* = \tfrac{1}{2}\epsilon + O(\epsilon^2). \tag{47}$$

The behaviour of $\tilde{\sigma}(t)$ can be found in a similar way. For time arguments $t > 0$ we can adopt the renormalization of $\phi^{(s)}(t)$ from the equilibrium theory, and a dissipation fluctuation theorem [9] yields

$$\phi^{(s)} \to \mathring{\phi}^{(s)} = Z_s^{1/2}\phi^{(s)} \,. \tag{48}$$

The response operator $\phi_0^{(s)}$ fixed to $t = 0$ requires a different renormalization because the field $\tilde{s}_0$ included in $\phi_0^{(s)}$ generates additional divergences. Furthermore, the second contribution to $\phi^{(s)}$ in (16) is suppressed for $t = 0$ due to the sharp Dirichlet initial condition $s_0 = 0$, and we get

$$\phi_0^{(s)} \to \mathring{\phi}_0^{(s)} = \mathring{\lambda}\mathring{\tilde{s}}_0 = Z_\lambda(Z_s Z_0)^{1/2}\phi_0^{(s)} \,. \tag{49}$$

The RGE for the coefficient $\tilde{\sigma}(t)$ which follows from (48) and (49) gives at the fixed point

$$\tilde{\sigma}(t) \sim (\mu^2\lambda t)^{(x_\lambda+\eta_0/2)/z} \,. \tag{50}$$

We can now exploit the results from the short time expansion to obtain the scaling behaviour of the correlation function $C(\boldsymbol{r}; t, t')$ and the response function $\chi_s(\boldsymbol{r}; t, t')$ for $t' \to 0$. As long as $t, t' > 0$ these functions satisfy the same scaling laws as in equilibrium. According to equations (46, 50) they vanish for $t' \to 0$ like powers of t', and we find

$$\begin{aligned} C(\boldsymbol{r}; t, t') &= r^{-(d-2+\eta)}\left(\frac{t}{t'}\right)^{\theta-1} g_C\left(\frac{r}{\xi}, \frac{t}{\xi^z}\right) \\ \chi_s(\boldsymbol{r}; t, t') &= r^{-(d-2+\eta+z)}\left(\frac{t}{t'}\right)^{\theta} g_\chi\left(\frac{r}{\xi}, \frac{t}{\xi^z}\right) \end{aligned} \qquad \text{for } t' \to 0 \tag{51}$$

with

$$\theta = -(1/z)(x_\lambda + \tfrac{1}{2}\eta_0)\,. \tag{52}$$

The functions $g_C(x, y)$ and $g_\chi(x, y)$ are the (finite) limits of $f_C(x, y, z)$ and $f_\chi(x, y, z)$ in (1, 2) for $z \to 0$.

So far we have only considered the response of the order parameter to a conjugate external field. In order to study the linear response to an external field coupling to the conserved densities we have to renormalize the composite response operator $\phi_j^{(m)} = -\lambda(\rho\Delta\tilde{m}_j + f\, l_{j,\alpha\beta}\tilde{s}_\alpha s_\beta)$.

For $t > 0$ a dissipation fluctuation theorem [9] yields $\mathring{\phi}^{(m)} = \phi^{(m)}$, i.e. $\phi^{(m)}$ remains unrenormalized, whereas for $t = 0$

$$\phi_0^{(m)} \to \mathring{\phi}_0^{(m)} = -\mathring{\lambda}\mathring{\rho}\Delta\mathring{\tilde{m}}_0 = Z_\lambda Z_\rho \phi_0^{(m)}\,. \tag{53}$$

A short-time expansion

$$\phi^{(m)}(t) = \tilde{\sigma}_m(t)\phi_0^{(m)} + \cdots \qquad \text{for } t \to 0 \tag{54}$$

analogous to (43) can be used to derive the scaling form

$$\chi_m(\boldsymbol{r}; t, t'; \tau) = \left(\frac{t}{t'}\right)^{\theta_m} r^{-d-z} f_\chi^{(m)}\left(\frac{r}{\xi}, \frac{t}{\xi^z}, \frac{t'}{t}\right) \tag{55}$$

for the m-response function, where the scaling function $f_\chi^{(m)}$ is finite for $t' \to 0$, and

$$\theta_m = -\frac{1}{z}(\gamma_\lambda^* + \gamma_\rho^*) = -\frac{4-d}{d} \tag{56}$$

is an universal exponent.

We conclude this section with a remark on the weak scaling fixed point

$$\rho_* = \infty \qquad w_* = \left(\frac{v^2}{\rho}\right)_* = \frac{2\epsilon}{4p+n} + O(\epsilon^2) \tag{57}$$

which is stable for $n > 4p + O(\epsilon)$. In the limit $\rho_* \to \infty$ the exponent η_0 becomes independent of the initial correlation c_0 and we have

$$\eta_0 = -\frac{n+2}{n+8}\epsilon + \frac{2p\epsilon}{4p+n} + O(\epsilon^2)\,. \tag{58}$$

In order to show that this universality holds to all orders in ϵ one can integrate the fields $\tilde{m}$ and m out from the statistical weight $\exp(-\mathcal{J} - \mathcal{H}^{(l)})$ and then perform the limit $\rho \to \infty$ at fixed $w = (f^2/\rho)$. In this limit all contributions corresponding to the non-equilibrium part (26) of the m-correlator vanish, and we obtain a new effective weight which is independent of c_0.

5. Conserved order parameter

As has already been pointed out in [3], time naively scales as $t \sim (\lambda\mu^4)^{-1}$ if the order parameter itself is a conserved density. Therefore the degree of divergence of integrals with broken translational invariance with respect to time is reduced by four, and no additional renormalization of the initial response field $\tilde{s}_0$ is necessary. For this reason the exponent η_0 is zero. Systems with conserved order parameter nevertheless show a non-trivial short time scaling behaviour governed by an exponent $\theta < 0$ if reversible drift terms are present in the Langevin equations. In this case equation (2) describes the growth of the reversible part of the response (which is absent at the beginning of the relaxation) as the system tends to equilibrium.

To give an example we consider the isotropic Heisenberg ferromagnet (model J [5]) with the dynamic functional

$$\mathcal{J}[\tilde{s}, s; \tilde{m}, m] = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d r \, \{\tilde{s}\partial_t s - \lambda(\nabla\tilde{s})^2 + \lambda(\nabla\tilde{s}_\alpha)[\nabla(\tau s - \Delta s + (g/3!)s^3)_\alpha + f\epsilon_{\alpha\beta\gamma}\nabla s_\beta s_\gamma]\} \,. \tag{59}$$

The linear response to an external field is given by

$$\chi(\boldsymbol{r} - \boldsymbol{r}'; t, t') = \langle s(\boldsymbol{r}, t)\phi(\boldsymbol{r}', t')\rangle \tag{60}$$

with

$$\phi_\alpha = -\lambda(\Delta\tilde{s}_\alpha + f\epsilon_{\alpha\beta\gamma}\tilde{s}_\beta s_\gamma) \,. \tag{61}$$

Now we proceed in the same way as in section 4. The renormalization of the composite response operator $\phi(t)$ depends on its time argument. For $t > 0$ we have $\phi \to \mathring{\phi} = Z_s^{1/2}\phi$ as in equilibrium while

$$\mathring{\phi}_0 = \mathring{\lambda}\mathring{\tilde{s}}_0 = Z_\lambda Z_s^{1/2}\phi_0 \,. \tag{62}$$

The different renormalizations of ϕ and $\phi|_{t=0}$ imply different anomalous dimensions. A short distance expansion yields the scaling behaviour (2) with the exponent

$$\theta = -\frac{4 - \eta - z}{z} = -\frac{6 - d - \eta}{d + 2 - \eta} \,. \tag{63}$$

6. Conclusions

Motivated by earlier studies of the non-equilibrium critical relaxation of models A and C which is governed by new, universal exponents we have considered the effect of reversible mode coupling on the short time scaling behaviour.

We have shown that (short-range) initial correlations of conserved densities coupling reversibly to the order parameter are not irrelevant. The width of the initial distribution of the conserved densities corresponds to a marginal variable which at the strong scaling fixed point enters into the value of the initial slip exponent θ, i.e. θ is not universal. For the response function of the conserved fields a short time scaling form with an universal exponent has been found.

Finally, it has been shown that the relaxation of the isotropic Heisenberg ferromagnet displays a short time scaling behaviour which is governed by a non-trivial initial slip exponent. We have expressed this exponent in terms of η and z.

An experimental test of these results by a quench of a real system from a high temperature to T_c is difficult since the temperature has to be stabilized very rapidly to render an observation of the initial stage of the relaxation possible. Therefore it would be worthwhile to look for the non-universal short time scaling behaviour in simulations [12]. However, there is no work known to the authors where the dynamics of a model with reversible mode coupling is studied by Monte Carlo techniques.

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